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Revisiting the Isochron Age Model

PART 1

article highlights

- Rock materials are dated according to the decay rate of certain radioactive isotopes.
- An isochron is a line on an isotope ratio diagram of rock samples.
- The y-intercept of the isochron line provides the ratio of the daughter isotopes when the rock first formed, and the line's slope supposedly provides an age.
- But a well-known "mixing problem" appears to give spurious results and therefore nullifies the isochron model as an accurate dating method.

Radioactive dating is based on the decay rate of a starting radioactive isotope (the parent) into its stable counterpart (the daughter). An age is assigned to an object by measuring the quantity of each isotope and calculating how long it would take for the parent to decay into the daughter. Since the mid-20th century, the isochron age model has been the standard for dating rocks, minerals, and crystals via the decay of certain radioisotopes they contain.

This model had its origins in a rather obtuse paper published in 1960.[1] Ironically, even the authors of this paper admitted that the potassium feldspars from the granitic rocks they analyzed gave a wide range of supposed ages. It's widely claimed that this model eliminates the need for any assumptions about the initial amount of the daughter isotopes when dating an object using the decay of specific isotopes within that object. But does it?

In the next two articles we'll take a closer look at the isochron age model and evaluate its ability to accurately assess the absolute historical age of an object. We will use the Rb-Sr (rubidium-strontium) decay pair for demonstration since it employs the least number of secondary assumptions among the various decay pairs used in radioisotope dating.

The Basics of the Isochron Age Model

The derivation of the mathematical framework for the general isochron age model begins with the basic radioisotope decay equation:

$$P = P_0 \times e^{-\lambda t_a} \tag{1}$$

Let's define some symbols:

$P \equiv$ Number of parent isotopes present at the time of measurement

$t_a \equiv$ Time from solidification of the measured crystal or rock suite to the present time, or the initial time of formation (t_0) to the final time of measurement (t_f), i.e., $t_a = (t_f - t_0)$

$P_0 \equiv$ Initial number of parent isotopes when decay begins

$D \equiv$ Total number of daughter isotopes present at time t_a

$D_0 \equiv$ Number of daughter isotopes present at t_0, or when the crystal/rock suite formed

$D_a \equiv$ Number of daughter isotopes added to the crystal/rock suite via decay of the parent

$P_r \equiv$ Number of parent nuclei that have decayed since the time of crystal/rock suite formation

$\xi \equiv$ The fraction of decays ($0 < \xi \leq 1$) that actually results in the daughter isotope of interest. This number is called the *branching ratio.*

In a closed system:

$$D_a = \xi \cdot P_r \tag{2}$$

and

$$P_r = P_0 - P \tag{3}$$

A closed system is one in which no parent or daughter atoms can escape or enter from outside. If the rock suite, mineral, or crystal exists as a closed system for millions of years, then no daughter atoms can leave or enter the system. This means any increase in the number of daughter atoms can *only* be due to radioactive decay:

$$D = D_0 + D_a \tag{4}$$

Then, subtracting D_0 from both sides of Equation (4) and substituting for D_a and P_r:

$$D - D_0 = \xi \cdot (P_0 - P) \tag{5}$$

Adding D_0 to both sides:

$$D = D_0 + \xi \cdot (P_0 - P) \tag{6}$$

Substituting for P_0:

$$D = D_0 + \xi \cdot (P e^{\lambda t_a} - P) \tag{7}$$

Then:

$$D = D_0 + P\, \xi \cdot (e^{\lambda t_a} - 1) \tag{8}$$

If we divide both sides of the equation by a stable index isotope (D_i) from the daughter's isotopic family (i.e., one that has the same number of protons but a different number of neutrons), then we have the basic isochron dating model equation:

$$\frac{D}{D_i} = \frac{D_0}{D_i} + \frac{P}{D_i} \cdot \xi \,(e^{\lambda t_a} - 1) \tag{9}$$

The equation for a straight line is:

$$y = mx + b \tag{10}$$

Here, m is the slope, a measure of the steepness of the line, and b is the y-intercept, the value of y when x equals 0. Note that if we treat D/D_i as the dependent variable y, P/D_i as the independent variable x, and D_0/D_i as the y-intercept b, then Equation (9) is also an equation for a straight line. This is a little easier to see if we rearrange the terms on the right-hand side of Equation (9) to give:

$$\frac{D}{D_i} = \xi \,(e^{\lambda t_a} - 1) \frac{P}{D_i} + \frac{D_0}{D_i} \tag{11}$$

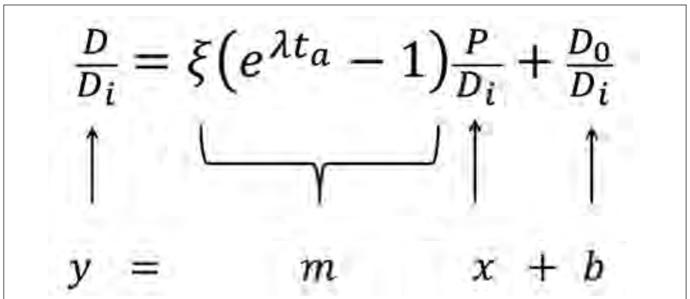

***Figure 1.** Equation (11) is actually the equation for a straight line if one takes* y *to be D/D_i,* x *to be P/D_i, and the y-intercept* b *to be D_0/D_i. In that case, the slope* m *of the straight line is $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_2}$. Figure created by Jake Hebert.*

Here, the slope m is equivalent to the factor in "front of" P/D_i in Equation (11). (See Figure 1.) Traditionally, the above equation is viewed as a linear equation in the isotope ratio variables $y = \frac{D}{D_i}$ and $x = \frac{P}{D_i}$. With this view in hand, one can plot the two isotope ratios on a linear graph with the dependent variable $y = \frac{D}{D_i}$ and the independent variable $x = \frac{P}{D_i}$ and then measure the slope of the resulting straight line ($m = \frac{\Delta y}{\Delta x}$); where $\Delta y \equiv (y_2 - y_1)$ and $\Delta x \equiv (x_2 - x_1)$. When calculating the slope, the points (x_1, y_1) and

(x_2, y_2) should be far apart and all data points should be used.[2] Scientists use a computer program to find the best-fit straight line through the data points and to find the slope of this best-fit line. It would seem to be a straightforward action to set the slope (m) equal to the multipliers of the independent variable *x*.

Note there is an additional factor (ξ) in this equation that's not normally seen in many textbooks, although Faure does make allowance for it if ^{40}K decay to ^{40}Ar is being used.[3] Its inclusion is usually not an important point; since most of the radiometric decay pairs used in dating the branching ratio are so close to 1, its inclusion in the model equation is moot.

By assumption, at the time, $t = 0$, that the lava or magma cools and hardens, the relative abundance of the daughter isotope, D_0/D_i, is the same everywhere in the lava, although the values of P_0/D_i can vary. This means that, in theory, the data points should form a flat, horizontal line at $t = 0$. This horizontal line is $y = b = D_0/D_i$. This problem manifests itself in the dating model because a second independent variable, i.e., time (t), has been inserted into the equation for a constant. However, as time passes, some of the parent atoms will decay into daughter atoms and project as an evolving straight line with decreasing amounts of the parent nuclei and increasing amounts of the daughter nuclei. (See Figure 2.)

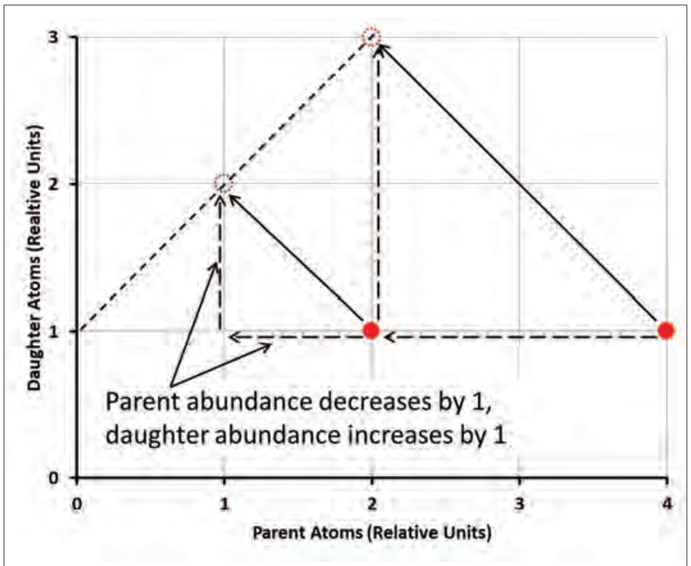

Figure 2. *Diagram illustrating the theory behind the isochron method. At time t = 0, the values of relative daughter/parent abundances should lie on a horizontal line. As illustrated by the leftmost red dot, when the abundance of parent atoms decreases from 2 to 1 due to radioactive decay, the abundance of daughter atoms must increase from 1 to 2 since the total number of atoms (parent plus daughter) remains the same. This causes the leftmost red dot to move diagonally up and to the left. One key aspect of radioactive decay and a consequence of Equation (1) is that the number of decays occurring in a given amount of time is directly related to the number of parent atoms. Because the starting abundance of parent atoms for the rightmost red dot is twice as large as the leftmost dot (4 as opposed to 2), the rightmost sample will experience twice as many decays (2 as opposed to 1) in the same amount of time. This causes the rightmost red dot to move left 2 units and up 2 units. The net effect is that the red dots continue to lie on a straight line but the slope of that line increases with time. In theory, the slope of this line should give the age of the sample, per Equation (11). This figure is based on Figure 6.3 in Steve Austin's* Grand Canyon: Monument to Catastrophe *and adapted for use by Jake Hebert.*[4]

In a closed system, this means that when the parent isotope decays, the parent to index daughter isotope ratio (x) must decrease, and simultaneously the radioactively produced daughter isotope to index daughter (y) must increase by the same amount. Secular geologists interpret an isochron's positive slope as a reflection of radioactive decay. However, this pattern can also be explained by an isotope mixing model.

For the moment let's ignore these distinctions and proceed to how the above equation is used to produce an age for a group of crystals or rock samples. Setting the multiplier of the parent to index daughter isotope ratio equal to the slope for a linear relationship, we have:

$$m = \xi \cdot (e^{\lambda t_a} - 1) \qquad (12)$$

Solving for t_a:

$$t_a = \frac{1}{\lambda} \ln \left(\frac{m}{\xi} + 1\right) \qquad (13)$$

Model Implications

This is the basic equation used to estimate ages with the isochron age model. Note that nowhere in the analysis has t_a been treated as a variable in the linear equation from which it was derived. Time in this equation is essentially a derived quantity in that what's actually measured are the ratios for the parent and daughter isotopes relative to an index isotope in a given sample from which the slope and intercept of the model are determined. Also note that for any noticeable slope, the age equation almost *guarantees* deep time because the $\frac{1}{\lambda}$ term completely dominates the ln term in the age equation. Figure 3 demonstrates an actual isochron of data from the Bass Rapids diabase sill in the Grand Canyon, with notations illustrating the relation of Equation (11) to a straight line.[5]

A recent analysis of the isochron dating model was done by Robert B. Hayes in the periodical *Nuclear Technology*.[6] He observes that the effects of differential isotopic mass diffusion aren't taken into account in the isochron age model and that this can yield spurious results. He concludes that the most rigorous method to mitigate iso-

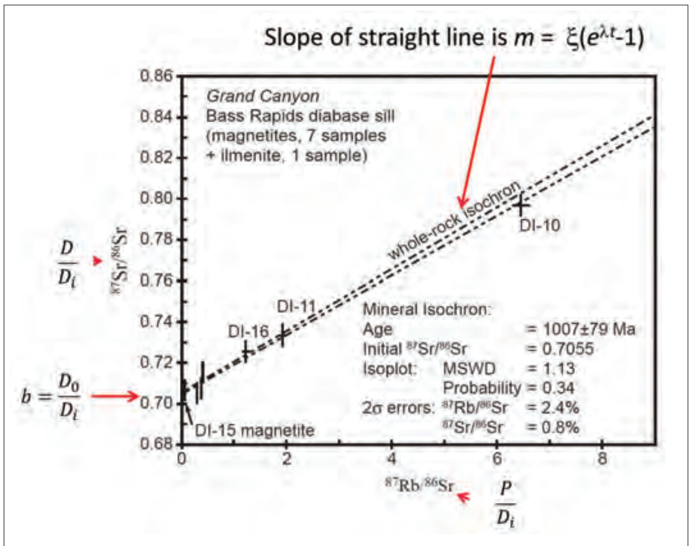

Figure 3. *A rubidium-strontium isochron graph showing the relationships between Equation (11) and the equation for a straight line. This graph is a ^{87}Rb-^{87}Sr analysis of mineral samples from the Grand Canyon Bass Rapids diabase sill. Data taken from the RATE project.*[5]

topic mass diffusion in dating applications, especially over geological time frames, is to not utilize isotopic ratios at all.

The only variables in the isochron dating models that are affected by the differential diffusion mechanism are the daughter and parent isotopes, which Hayes discusses in his paper. Aside from the problem of differential isotopic mass diffusion, there are also the problems of possible hydrothermal transport, isotope mixing, and fractionation processes that need to be considered. There's also the fact that the method fails to take into account past accelerated nuclear decay.[7]

Conclusion

So, what can be said about the isochron age model? It appears more like a mixing model than a model clock. Gunter Faure points out[8] that a study of strontium isotopes contained in alkalic rocks from the Birunga and Toro-Ankole regions of equatorial Africa produced a fictitious isochron.[9] Y.-F. Zheng stated:

> It is clear that mixing of pre-existent materials will yield a linear array of isotopic ratios. We need not assume that the isotopes, assumed to be daughter isotopes, were in fact produced in the rock by radioactive decay. Thus, the assumption of immense ages has not been proven. The straight lines, which seem to make radiometric data meaningful, are easily interpreted to be the result of simple mixing.[10]

The mainstream geological community still continues to treat the results from isochron model dating as absolute scientific fact, but it's been clearly known for 30 years there are unresolved problems with the model. The more analysis is done on the iconic isochron model, the more dubious it appears.

References

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